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A Comprehensive Review of Vehicle and Road Condition Estimation Techniques

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Abstract

This article reviews the key physical parameters that need to be estimated and identified during vehicle operation, focusing on two key areas: vehicle state estimation and road condition identification. In the vehicle state estimation section, parameters such as longitudinal vehicle speed, sideslip angle, and roll angle are discussed, which are critical for accurately monitoring road conditions and implementing advanced vehicle control systems. On the other hand, the road condition identification section focuses on methods for estimating the tire–road friction coefficient (TRFC), road roughness, and road gradient. The article first reviews a variety of methods for estimating TRFC, ranging from direct sensor measurements to complex models based on vehicle dynamics. Regarding road roughness estimation, the article analyzes traditional methods and emerging data-driven approaches, focusing on their impact on vehicle performance and passenger comfort. In the section on road gradient estimation, details are given on how to measure the grade and bank angles of a road, and their role in enhancing vehicle stability under extreme driving conditions is emphasized. The article also provides an in-depth overview of different vehicle state estimation techniques, including model-based, observer-based, and techniques using neural networks for estimation. Finally, the article summarizes the challenges facing current research and suggests potential directions for further research. The article emphasizes the importance of combining vehicle state estimation with road condition recognition and suggests that this combination has the potential to provide a more robust framework for adaptive vehicle control systems in variable and complex driving environments.

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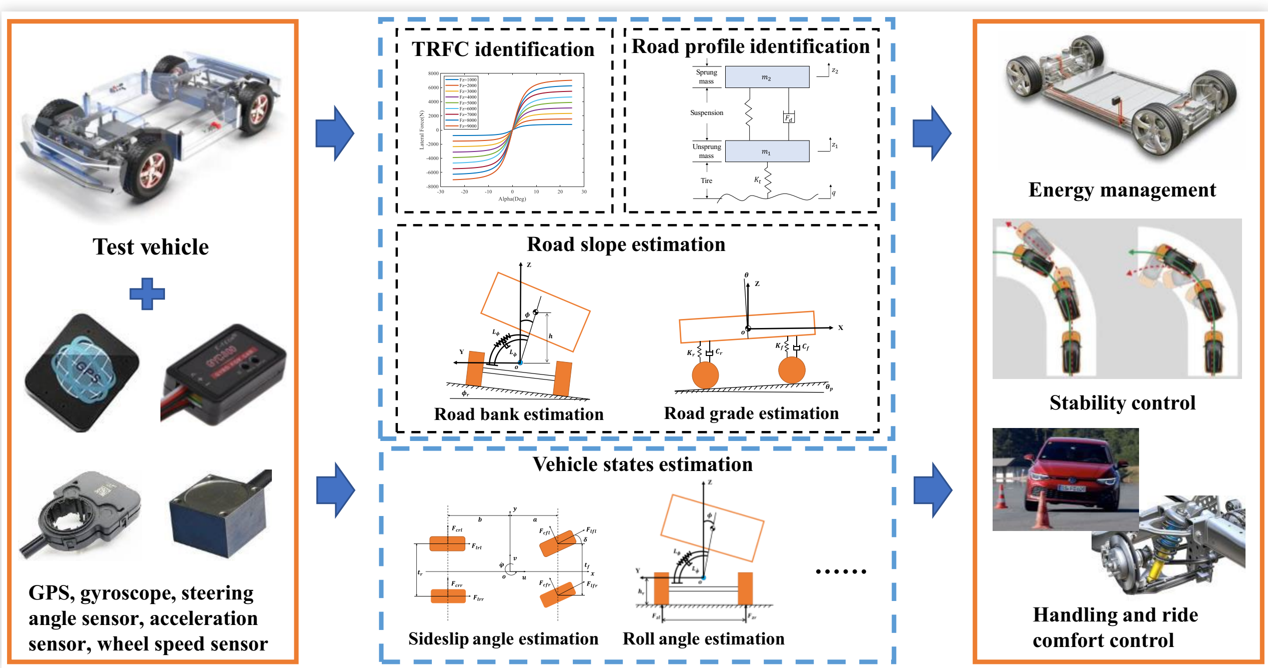
1. Introduction

Accurate estimation of vehicle states and road conditions, which are the basis of vehicle safety, performance, and proper operation of advanced driving assistance systems (ADAS) [1], requires that the technology advances a great deal. The necessity for precise estimation of these variables has been emphasized due to the rapid progress of autonomous driving technologies. Dynamic vehicle parameters such as longitudinal and lateral speeds, roll angle, and sideslip angle are measured during vehicle state estimation. Controlling these parameters is needed for the suitable control command of the vehicle. This is why systems such as the anti-lock braking system (ABS), which is one of the representative systems to ensure safety, need this data to ensure that the vehicle remains within the stipulated safety and reliability brackets under different driving conditions. Another key information is the condition of the road. Road condition parameter estimation involves the detection and estimation of TRFC, the roughness of the road, and the gradient of the road. These aspects have some bearing on the way the vehicle can be steered, driven, and felt within. The performance of the road condition estimation must be high to achieve the best control strategy and for the active safe system enhancement [2].

Based on the theoretical framework shown in Figure 1, this study further systematically explains the latest research progress in vehicle state and road parameter estimation methods. The stable operation of the vehicle system and its excellent driving performance are the result of the synergy between the vehicle and road estimation systems. The vehicle state estimation module constructs dynamic control benchmarks by acquiring sensor data, while the road estimation module provides environment-aware inputs by recognizing TRFC and road geometry in real time. The data fusion between the two enables the control system to realize autonomous cognition of the vehicle itself and the road based on multi-source sensor information, especially in complex and harsh driving environments.

Vehicle state estimation is one of the core elements of autonomous driving technology, which is of great significance for building future intelligent transportation systems. Its core function lies in the real-time acquisition of vehicle dynamic data, including key parameters such as longitudinal velocity, lateral velocity, roll angle, and sideslip angle, as well as the monitoring of the interaction force between the vehicle tire and the road surface. An accurate vehicle state estimation system is crucial for ensuring driving safety and improving operational efficiency, and it provides important data support for ADAS such as adaptive cruise control (ACC), automatic parking system, and collision warning system. Vehicle state estimation systems can be divided into three main categories: the first is a sensor-based approach, which collects vehicle information by integrating devices such as GPS, inertial measurement units (IMU), and wheel speed sensors; the second is a dynamic model-based approach, typically represented by techniques such as Kalman filtering and sliding mode observers; and the third is a data-driven approach, which mainly employs artificial intelligence techniques such as machine learning and deep learning. Each of these methods has its characteristics, and their applicability and effectiveness vary according to specific application

FIGURE 1 Estimation and identification schematic.



scenarios. In practical applications, it is often necessary to organically integrate multiple estimation techniques to achieve more accurate vehicle state estimation. Such accurate estimation is not only the basis for realizing autonomous driving, but also the key to further improving the performance of the autonomous driving system.

Accurate identification of the TRFC is critical to the precise estimation of road conditions, as it directly affects vehicle stability, braking performance, and overall maneuverability. Current TRFC estimation techniques, including model-based observers and sensor fusion methods, are important tools for improving the performance of critical systems such as ABS and electromechanical braking (EMB) systems. Accurate estimation of TRFC is essential to ensure that these systems perform optimally under different vehicle operating conditions [3]. TRFC estimation methods fall into two main categories. The first category is direct measurement techniques, which are based on stopping distances and measuring the actual friction level between the tire and the road surface via optical sensors or accelerometers. These methods can provide real-time data, but the measurements are susceptible to interference from environmental factors, which may lead to estimation errors. The second type is the model-based observation method, in which the TRFC is derived from a vehicle dynamics model. This method is more stable than direct measurement and can provide consistent estimation results under different driving conditions, but its computational complexity is high, and it relies on accurate mathematical models and algorithm settings. To improve the estimation accuracy, the mapping relationship of TRFC-related parameters is usually constructed using sensor data in conjunction with an experimental strategy. For example, acoustic and temperature sensors are being investigated to improve the perception and estimation of tire–pavement friction characteristics. Traditionally, model-based approaches have been the mainstay of TRFC estimation, thus employing complex mathematical models of tire and vehicle dynamics, such as wheel motion models, body motion models, or tire models (e.g., magic formula or dugoff models). In recent years, with the rapid development of artificial intelligence (AI) technology, there has been a new trend of combining AI with traditional model-based approaches. AI enables these models to adaptively adjust to different road environments and changes in vehicle dynamics, thereby improving the accuracy and applicability of TRFC predictions. This combination not only enhances the reliability in practical applications, but also further validates the superiority of the method, making the TRFC estimation more efficient and closer to the actual working conditions.

Evaluating pavement roughness is a key prerequisite for studying pavement defects, as it directly affects the performance of the suspension system and vehicle ride quality. Accurate estimation of roughness not only helps to optimize the suspension system and improve the stability of the vehicle, but also reduces the wear and tear of key components, thus extending the service life of the vehicle. Currently, road surface profile information can be effectively obtained by frequency domain analysis and machine learning models. These two methods have their advantages and disadvantages in terms of computational accuracy and computational complexity. Road surface roughness and slope are important factors affecting the adjustment of vehicle dynamics, which have a direct impact on driving comfort, key vehicle kinematic parameters, and control strategies. Sensors such as accelerometers and LiDAR (laser radar) are an important part of the intelligent vehicle control system, which can monitor the road surface conditions in real time and adjust the suspension system to adapt the vehicle to the ever-changing road surface environment, thus improving driving safety and ride comfort [4]. Accurate perception of complex road surfaces is one of the basic conditions for optimizing vehicle motion control systems. In recent years, advances in sensor technology and optimization of machine learning algorithms have led to more accurate road roughness estimation. These technological developments have driven real-time adaptive suspension systems that not only improve the overall performance of the vehicle, but also ensure stable handling under different road conditions [5]. In modern complex traffic environments, access to real-time road surface roughness data is critical. Using this data, vehicles can quickly adjust handling strategies to maintain stability and optimize the driving experience on poor or polluted surfaces. Therefore, accurate road roughness recognition is not only a key factor in improving vehicle safety and comfort, but also an important direction for the future development of intelligent transportation systems [6].

Vehicle control systems need to accurately estimate road slope information, including grade and bank angles, which is critical to improving energy efficiency and maintaining stability, especially when traveling up and down hills or with minor road surface changes. Correct road slope information optimizes traction control, torque distribution, and braking, resulting in more efficient vehicle operation under different operating conditions, lower energy consumption, and improved driving safety. In addition, accurately recognizing road slope is important for predictive control systems such as HSA, ACC, and electronic stability control (ESC). ADAS relies on road grade angle data to make real-time adjustments that enable vehicles to maintain greater stability, safety, and overall performance on rough roads [7]. Since road slope has a significant impact on vehicle dynamics, accurate road slope estimation not only helps optimize energy consumption, but also affects the ease of driving maneuvers. Therefore, incorporating high-precision slope angle estimation in the controller allows the vehicle to cope with complex road conditions more effectively, optimize energy consumption, and enhance driving safety. This approach is in line with the concepts of green driving, eco-driving, and safe driving, and enables vehicles to realize smarter and more environmentally friendly operation modes under different road conditions [8].

This study comprehensively explores the cutting-edge techniques of vehicle state estimation and pavement state recognition and thoroughly analyzes the practical applications of these methods and their possibilities of mutual integration. The research is presented systematically so that the roles of different techniques in vehicle dynamics analysis can be demonstrated. First, the research focuses on road state estimation, exploring effective methods for predicting the TRFC, roughness, and road slope. Then, the research shifts to vehicle state estimation, covering traditional sensor-based methods, dynamics model-based methods, and modern data-driven techniques. This review systematically analyzes the advantages and disadvantages of each type of technique under different driving scenarios and emphasizes the impact of road surface conditions on vehicle dynamics and handling stability, revealing the strong link between vehicle efficiency and road conditions. In addition, this study proposes a framework that integrates vehicle state and road condition estimation, aiming to bridge the technology gap in current research. This fusion approach can provide enhanced safety and optimize vehicle handling, comfort, and driving experience under complex or adverse driving conditions. Finally, the study looks at future directions and proposes key measures to improve vehicle state and road state estimation, laying the foundation for smarter and safer vehicles.

2. Tire–Road Friction Coefficient

Accurate estimation of the TRFC is critical for improving vehicle safety systems, such as ABS and ESC. Recent advancements, particularly in sensor fusion and Kalman filtering techniques, have significantly enhanced TRFC estimation, reducing weather-related accidents and improving overall vehicle dynamics. The role of TRFC is central to vehicle stability, influencing the effectiveness of traction control and ABS systems [9–12]. Recent research has focused on developing advanced algorithms, including machine learning models, adaptive Kalman filters, and observer-based estimators, to improve TRFC estimation. These techniques tackle key challenges such as noise reduction, real-time processing, and adaptability to varying driving conditions. TRFC estimation methods are typically categorized into two approaches: off-board sensor-based methods, which rely on direct measurements from sensors such as optical and acoustic devices, and vehicle dynamics model-based methods, which estimate friction by analyzing vehicle responses.

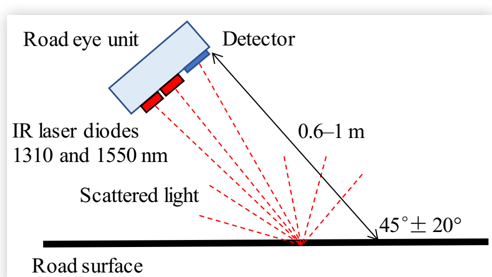
Sensor-based methods offer high accuracy in controlled environments, while model-based methods are more adaptable to different road conditions, though they often require complex calibration and data fusion techniques. The continuous exploration and refinement of these methodologies highlight ongoing efforts to enhance vehicle safety and performance across diverse driving environments [13, 14]. These advancements not only help reduce weather-related accidents but also drive progress in automotive safety technologies and road safety standards.

2.1. Off-Board Sensor Experiment-Based

Experiment-based methods are linked to TRFC through information provided by high-precision sensors such as infrared, ultrasonic, and multispectral cameras. In recent years, breakthroughs in interoperable sensor technologies have significantly improved the accuracy and robustness of these methods under complex and variable operating conditions [15]. For example, sensor inputs such as vibration signals, temperature variations, and the like are valuable for analyzing road–tire interactions [16]. In addition, current sensor fusion techniques are moving toward the integration of multimodal data streams, an approach that not only improves the accuracy of friction estimation algorithms, but also enhances their adaptability and robustness under different driving conditions [17]. By combining data from multiple sensors, TRFC estimation can be more accurate and provide more reliable information support for vehicle control systems.

2.1.1. Optical Sensors and Cameras Optical sensors, cameras, and ultrasonic technology have great potential for TRFC estimation performance. Optical sensors offer significant advantages in detecting structures and materials on road or sidewalk surfaces, while cameras analyze road conditions by evaluating the texture and reflectivity of the road surface. On the other hand, ultrasonic sensors rely on virtual distance measurement and surface mismatch detection to sense roadway features. The combination of these sensors provides a reliable basis for data fusion and makes friction estimation more accurate, despite the need to operate in well-lit conditions and minimize ambient noise interference. Optical sensors exhibit extremely high sensitivity to multiple wavelength reflectance characteristics in the infrared band, allowing them to differentiate between dry, slippery, icy, or snow-covered road surfaces. Figure 2 illustrates a schematic of

FIGURE 2 Road–eye sensor schematic.



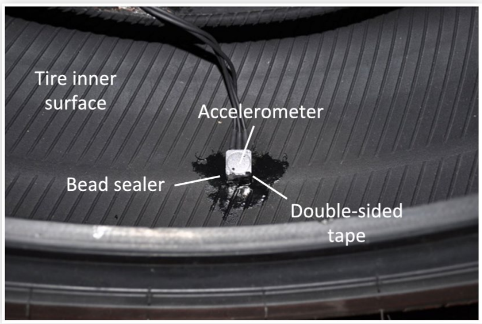
a road–eye sensor to illustrate the application of optical sensors in accurately characterizing road surfaces. Additionally, optical sensors can be used to assess the dynamic deformation of tire sidewalls and correlate it with friction dynamic characteristics [15]. The combination of cameras and optical sensors enables the capture of finer road surface information, which in turn improves the accuracy of friction estimation [16]. Since the magnitude of friction is significantly affected by the characteristics of the pavement texture, after recognizing the pavement texture, neural networks can be trained to develop more accurate pavement friction recognition algorithms [17]. The development of this technology provides key support for smarter vehicle control systems and improves the stability and safety of vehicles under different road conditions [17].

According to an analysis of recent literature [18–20], the combination of optical sensors and image processing innovations enables the efficient extraction of the TRFC by targeting pavement texture and water level diagnosis. It has been found that the coupling of optical data and machine learning brings about higher levels of friction estimation accuracy, especially in changing environments such as wet, icy, or dry. Holzmann et al. [20] presented a camera and microphone friction coefficient estimation method. Their model analyzed brightness (luminance) and pixels captured on the pavement by the camera. The frequency range of the pavement sound is recorded by the microphone in the spectrum of 100–600 Hz. The data covered in the acoustic and visual categories of friction exists in this algorithm. Consequently, the accuracy of the calculation is improved. Kuno and Sugiura [18] implemented a CCD camera for recording road conditions by analyzing brightness dispersion to monitor the presence of water. This method works with a change in the luminance signal to differentiate between dry and wet road surfaces. Jokela et al. [19] worked on developing two different techniques for the detection of road conditions. The first method requires the measurement of the changes in polarization of light. It also involves the tinkering of the pavement condition using reflected light from both straight and tilted directions. The second method uses granularity analysis and applies a low-pass filter to blur the original image. Thus, the contrast ratio between the unfiltered image and the processed one helps with the larger contrast gain spectrum of road conditions. These methods highlight the use of optical sensors in friction coefficient estimation. Indeed, in relatively difficult situations, image processing and data analysis over the original images are likely to bring greater accuracy and reliability to the estimation.

The above research has significantly improved the flexibility of optical sensors and cameras to measure the coefficient of friction under different contrasting conditions and to differentiate between pavement types. However, the development of this technology still faces some challenges. Optical sensors have limitations in coping with light variations and complex weather conditions, and their response speed and stability directly affect the accuracy of measurements. To compensate for this, data from other sensors, such as ultrasonic, infrared, LiDAR, or IMUs, can be combined to enhance the robustness of the estimation through sensor fusion. By integrating data from multiple sources, it not only reduces the error of optical sensors, but also provides more accurate and reliable friction estimation under different environmental conditions, thus enhancing the safety and adaptability of vehicles in complex road conditions. Baffet et al. [21] presented the TRFC estimation model obtained artificially using a combination of multivariate linear regression and fuzzy logic techniques. Du et al. [22] put forth an approach involving a driving-wheel robot operating on a well-trained deep neural network. This contains domain-specific knowledge to boost prediction accuracy. Leng et al. [23] proposed an approach to vehicle friction recognition by developing a hybrid framework. This framework synthesized a dynamic estimator and visual estimator, which gives significant friction factor recognition. The use of a backpropagation (BP) neural network was implemented for the friction coefficient prediction by Yu et al. [24]. In this way, although it has high-accuracy performance in good vision cases, the accuracy suffers in low visibility conditions such as night driving or inclement weather. These studies indicate that the optical sensor is suitable for estimating friction. However, the limitations of optical sensors' sensitivity to environmental changes should be tackled by combining them with other sensors to deliver high-accuracy estimations and robust systems for applications.

2.1.2. Deformation and Vibration Method Recent studies have investigated several approaches to estimating the TRFC. These methods involve the coupling of tire deformation, vibrations, and friction levels. Such investigations unveiled significant relations between tire dynamics and TRFC. Studies have demonstrated that vibration and deformation analysis, as well as accelerometers fitted inside the tires, capture vital data. Inside tires, high-frequency vibration is measured with placed accelerometers. This represents the tire imperceptibly touching the road. Researchers increase the TRFC by monitoring this vibration through analyses. From tire deformation and resonance patterns, it is evident how the tires behave under different driving conditions, as shown in Figure 3. Apparently, Singh et al. [25] used vibration response frequency analysis to predict TRFC. By analyzing the influence of road characteristics on stiffness and damping parameters, the technology evaluates the road adhesion conditions from the perspective of the frequency domain. The method is particularly suitable for frequency response systems. A test study has shown that the use of superimposed sensors can significantly improve the accuracy of TRFC (road adhesion coefficient) estimates compared to traditional methods. These advanced systems can

FIGURE 3 Inner tire vibration sensor.



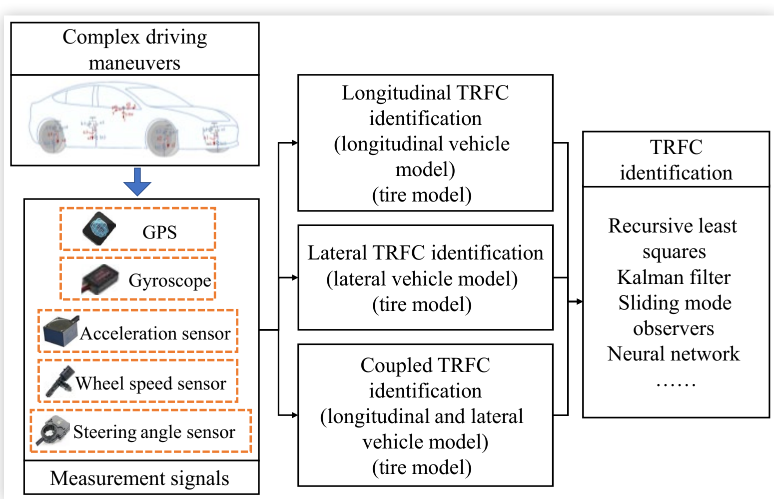
track tire deformation patterns and motion in real time, and detect and respond to subtle changes in the road surface in a very short time. In addition, they continuously monitor tire deformation and vibration patterns to accurately identify different road conditions [26–30]. However, while the sensor-based approach shows great potential, it still faces challenges in practical applications. For example, in harsh operating environments, the sensor may experience displacement errors or physical damage, affecting the measurement accuracy. Therefore, the focus of current technology is to develop more robust sensor designs and optimize data filtering and fusion algorithms to improve the robustness and adaptability of the system.

Along with the accelerometer, other techniques such as ultrasonic sensors [31], laser profilometers [32], wireless piezoelectric tire sensors [33], and magnetometers [34] were explored as well. These technologies provide additional information for friction estimation and quantification of surface irregularities. Hence, calibration of these systems is usually provided to specifically customize them for the dynamics of the vehicle. However, there are still many challenges to integrating offline sensors into production vehicles, such as the installation of additional components and the need for high-precision calibration. Current research is focused on developing a compact, integrated sensor system to enhance the road experience of the average driver without significantly increasing the cost of the vehicle. Although such systems are susceptible to TRFC distortion, they still offer a possibility to extend the measurement range. It is worth noting that in extreme driving conditions, such as rainy days or dusty environments, these factors have less of an impact on measurement quality. Although the research under controlled laboratory conditions can provide a certain reference value, its authenticity still has certain limitations in the real driving environment, resulting in experimental results that may be different from the actual situation. At present, there has been relatively little research into this difference. However, model-based approaches to vehicle dynamics have been adopted by numerous automakers and are considered a more reliable option. These object-oriented modeling methods not only consider the dynamic characteristics of the vehicle itself, but also combine environmental factors, to improve the reliability of vehicle state estimation.

2.2. Vehicle Dynamics Model-Based

Vehicle dynamics-based methods rely on longitudinal and lateral dynamic models of the vehicle, combined with precise tire models such as the magic formula or dugoff model. These models capture the complex interactions between tires and the road, essential for accurate TRFC estimation under varied driving conditions. Key vehicle states, including body acceleration, angular velocity, and wheel speed, are measured using sensors such as accelerometers, gyroscopes, and wheel speed sensors. These measurements feed into dynamic models, enhancing the accuracy of TRFC estimation by reflecting real-time vehicle–road interactions. By combining this information with the vehicle dynamic model, TRFC can be identified, as shown in Figure 4. Vehicle dynamic model-based

FIGURE 4 The general framework of TRFC identification.



identification methods include tire models (e.g., magic formula, dugoff), longitudinal dynamics models that estimate forces during acceleration and braking, lateral dynamics models for stability control, and coupled dynamics models that account for interactions between longitudinal and lateral forces, offering a holistic view of vehicle behavior on different surfaces.

2.2.1. Tire Models The tire is the only component of the vehicle in contact with the ground, and there is a close link between tire forces and tire slip, slip angle, and TRFC. A model that describes the tire in detail is essential. Various mathematical models for tires have been created, considering both the tire model itself and the time behavior it can represent, whether steady-state or transient (refer to Figure 5) [35]. This article introduces some of these models, specifically those more commonly employed for estimating friction.

1. Pacejka tire model:

The Pacejka tire model, also known as the magic formula, was initially proposed by Pacejka in 1992 [36]. This semi-empirical model employs specialized functions to depict longitudinal tire force, lateral tire forces, as well as aligning moments. The formulations in this tire model are as follows:

$$\begin{aligned} y &= D\sin\left(\left[C\arctan\left\{Bx - E\left(Bx - \arctan Bx\right)\right\}\right]\right) + S_v \\ x &= X + S_h \end{aligned} \tag{1}$$

where y is longitudinal force, lateral force, or aligning moment, X is tire slip angle or longitudinal slip rate, D is peak factor, C is shape factor, B is stiffness factor, E is curvature factor, S_v is horizontal drift, S_h is vertical drift.

The magic formula is widely used for vehicle state estimation and TRFC identification [36–38]. Kim et al. [39] adopted the conventional friction concept and presumed the coefficients of the magic formula to be known for formulating friction. Employing an instrumented vehicle, they inferred the forces acting on each wheel using the magic formula, subsequently estimating both longitudinal and lateral friction coefficients. On the other hand, Yi et al. [40] utilized an observer-based algorithm wherein the magic formula with predefined parameters was employed to compute longitudinal force and estimate the tire–road friction coefficient. Initially, they employed a sliding mode observer (measuring wheel angular velocity) to estimate vehicle states and then utilized a recursive least square algorithm to estimate the tire–road friction coefficient.

2. Dugoff tire model

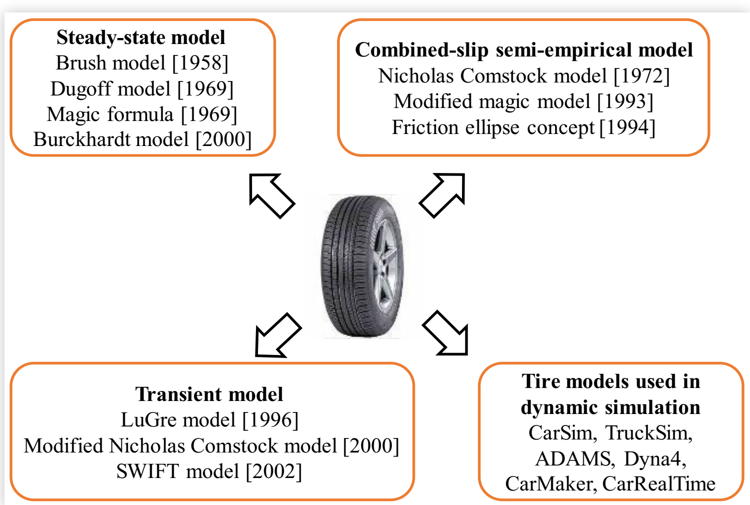
The dugoff tire model, devised by dugoff et al. in 1969 [41], is a physical model that assumes a uniform vertical pressure distribution across the tire contact patch. In its basic representation, this model describes the relationship between longitudinal and lateral forces and slip as a function of two key parameters: tire stiffness, which governs the slope of the force–slip curve at low slip values, and the tire–road friction coefficient, which dictates its curvature and peak value. The formulation of the dugoff model is as follows:

$$\begin{aligned} F_x &= \mu \cdot F_z \cdot C_x \cdot \frac{\lambda}{1-\lambda} \cdot f\left(L\right) \\ F_y &= \mu \cdot F_z \cdot C_y \cdot \frac{\tan\alpha}{1-\lambda} \cdot f\left(L\right) \end{aligned} \tag{2}$$

where μ is the road adhesion coefficient, α is the tire slip angle, λ is the tire slip ratio, C_x and C_y are tire longitudinal stiffness and cornering stiffness.

$$f\left(L\right) = \begin{cases} L\left(2-L\right), & L < 0 \\ 1, & L \geq 0 \end{cases}$$

FIGURE 5 Tire model.



$$L = \frac{1}{2\sqrt{C_x^2 \cdot \lambda^2 + C_y^2 \cdot \tan\alpha^2}} \cdot (1-\lambda) \cdot$$
$$\left(1 - \varepsilon \cdot v_x \cdot \sqrt{C_x^2 \cdot \lambda^2 + C_y^2 \cdot \tan\alpha^2}\right)$$

The tire slip angle is:

$$\alpha_{fl} = \delta - \arctan\frac{v + a/\gamma}{u + t_f \cdot \gamma / 2}$$
$$\alpha_{fr} = \delta - \arctan\frac{v + a/\gamma}{u - t_f \cdot \gamma / 2}$$
$$\alpha_{rl} = -\arctan\frac{v - b/\gamma}{u + t_r \cdot \gamma / 2}$$
$$\alpha_{rr} = -\arctan\frac{v - b/\gamma}{u - t_r \cdot \gamma / 2}$$

where t_f and t_r are the wheel track of the front and rear wheel, γ is the camber angle of each wheel.

The tire slip ratio is:

$$\lambda_{ij} = \begin{cases} \dfrac{\omega_{ij} \cdot R_e - u_{ij}}{\omega_{ij} \cdot R_e} > 0, & \text{Deceleration} \\ \dfrac{\omega_{ij} \cdot R_e - u_{ij}}{u_{ij}} < 0, & \text{Accleration} \end{cases}$$

where ω_{ij} is wheel speed, R_e is the wheel radius.

The dugoff tire model is currently the most commonly used model for TRFC identification, which has the advantage that TRFC is expressed in explicit form in the equation. At the same time, the dugoff tire model has two disadvantages of its own: there is no peak point and the maximum value is smaller than the magic formula; as the tire slip and slip angle increase, the difference between the longitudinal and lateral forces of the two models increases. Therefore, a correction coefficient is introduced to correct the model, and the correction can keep good agreement with the magic formula [42, 43]. Recent research often involves multi-step processes where tire forces (longitudinal, lateral) and slip rates are initially estimated using observer models or neural networks, followed by TRFC calculation through algorithms such as the dugoff or LuGre tire models. This sequential estimation approach enhances the adaptability of TRFC identification in real-time vehicle dynamics [44, 45].

3. LuGre tire model

The LuGre tire model, developed by De Wit et al. in 1995 [46], is a physics-based dynamic model that represents contact surfaces with elastic bristles. This model captures the complex friction dynamics between the tire and road by simulating the bristle deflection, providing a realistic representation of tire–road interactions crucial for accurate TRFC estimation. In the lumped LuGre model, the average deflection of these bristles (denoted by z) is expressed as:

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)} z$$
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$
$$\sigma_0 g(v) = F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2} \tag{3}$$

where v is the relative velocity between the two surfaces, v_s is Stribeck velocity, F_c is the Coulomb friction level, F_s is the level of stiction force, σ_0 is rubber stiffness, σ_1 is rubber damping coefficient, and σ_2 is the viscous relative damping. In the distributed LuGre model, an area of contact is assumed between the tire and the road, which formulates the friction force as follows [47]:

$$F(s) = \mathrm{sgn}(v_r) F_n g(s) \left(1 + \gamma \frac{g(s)}{\sigma_0 L |s|} \left(e^{-\frac{\sigma_0 L |s|}{g(s)}} - 1\right)\right) + F_n \sigma_2 r \omega s$$
$$\gamma = 1 - \sigma_1 |v_r| / g(s)$$
$$g(s) = \mu_c + (\mu_s + \mu_c) e^{-|r\omega s / v_s|^{\frac{1}{2}}} \tag{4}$$

where F_n is the normal load, L is the contact patch length, and $v_r = (r\omega - v)$ is the relative velocity.

The LuGre model is widely used for estimating friction forces and coefficients by simulating tire dynamics under different road conditions. Recent studies utilize the LuGre model in traction control systems and emergency braking scenarios, but its complexity and the need for parameter tuning remain significant challenges. De Wit et al. [47] used a single-wheel dynamic model combined with the lumped LuGre friction model, introducing a novel parameter that dynamically adjusts to road conditions, enhancing the model's ability to reflect real-time changes in TRFC estimation, particularly during variable surface interactions. By using wheel angular velocity data, De Wit et al. developed an online observer that significantly improved the accuracy of vehicle longitudinal velocity and road condition estimations. However, the method's accuracy can be affected by sensor noise, requiring advanced filtering techniques for optimal performance. Similarly, Alvarez et al. [48] applied the LuGre model in designing a tire friction estimation system for emergency braking control. This approach leveraged the LuGre model's ability to simulate dynamic friction accurately under sudden braking conditions, improving control system responsiveness and safety. Chen et al. developed an observer using a bicycle model to estimate the internal states of the LuGre tire model. By integrating a recursive least square algorithm, the approach improves the estimation of friction coefficients, making it more suitable for real-time applications in vehicle dynamics control. Alvarez et al. [48] further developed an adaptive friction estimation algorithm based on the LuGre model, employing a quarter-car model and a sliding mode observer to estimate vehicle velocity and internal LuGre model parameters. Matusko et al. [49] employed a lumped LuGre model combined with a neural network algorithm to better capture the complexities of friction force dynamics. The neural network component effectively compensates for model uncertainties, enhancing the LuGre model's predictive accuracy in variable driving conditions, adapted via the Lyapunov direct method. The LuGre tire model also serves as a basis for new dynamic models, such as Cleays et al. [50]. LuGre-based model, which extends the original framework to include both longitudinal and lateral forces as well as aligning moments. These enhancements make the model highly applicable to modern vehicle control systems, including traction and ABS controllers.

2.2.2. Longitudinal Dynamics Models Recognition methods based on longitudinal vehicle dynamics enhance the observability of wireless vehicle monitoring systems by using specially created inputs such as acceleration and braking. These movements show the force changes inside the tires. It is key in estimating the friction coefficient between the tire and the surface condition. The vehicle longitudinal dynamics field is mostly covered by the cooperation of a few related models. The first model describes the forward motion of the vehicle. The second one will look into the rotational dynamics of the wheels. The model used by the third one is a tire–road surface interaction forces analysis. Combining these three models, we shall have an overall perception of the relationship between the motion of the vehicle and the traction force of the tires.

Vehicle longitudinal motion model:

$$m\dot{u} = F_{x_{fl}} + F_{x_{fr}} + F_{x_{rl}} + F_{x_{rr}} \tag{5}$$

where m is the vehicle's total mass, u is longitudinal velocity, F_{xfl}, F_{xfr}, F_{xrl}, F_{xrr} are longitudinal tire force.

The longitudinal vehicle model describes the interaction between vehicle acceleration, tire force, and tire slip ratio. The model provides a connection between these parameters within the stated geometrical arrangement and equation of motion through the effect of slip ratio variation on the TRFC. Consequently, with the enhancement of this analysis, grip under different conditions such as acceleration or braking is being enriched. In the same vein, the demanded tire force extends with the increment of the slip ratio. But when the maximum tire force is achieved, it is observed that it lessens a bit. Just like in the previous case, TRFC could rise with the increasing slip angle, thus showing a similar trend. Gustafsson [51] suggested a new way of calculating the TRFC by measuring the number of lines on the standard curve, the curve included. Further, this curve exhibits a linear relationship, which facilitates the change in slip ratio to be identified with the change in tire force. This method allows determining the TRFC during a specific driving maneuver.

Wheel rotation model:

$$\begin{aligned} m_w\dot{v}_x &= F_x - F_{rr} \\ J_w\dot{\omega}_w &= \left(T_w - T_b\right) - F_x r_w - F_{rr} r_w \end{aligned} \tag{6}$$

where $\dot{m}_w$ is the wheel mass, F_x is the longitudinal tire force, F_{rr} is the wheel rolling resistance, J_w is the moment of inertia of the wheel, ω_w is the wheel speed, T_w and T_b are the drive and brake torques, r_w is the rolling radius of the tire.

Wheel dynamics can be employed together with detailed tire models to estimate longitudinal forces and friction using an analysis of the rolling behavior of the wheels under different load and slip conditions. This method is thought to be an accurate friction estimator during braking/acceleration processes, which are significant for vehicle control systems [52]. Hsiao et al. [53] proposed a method based on moment balance equations for each wheel obtained from the substitution of wheel torque and angular velocity into the equations. The developed model determines the longitudinal forces generated by the tires. Rajamani et al. [54] used the single-wheel model that employs the wheel dynamics together with transferring angular velocity data. Also, a slip mode observer is applied for the TRFC estimation. With this approach, both adaptability to diverse driving environments and improved TRFC estimation can be realized. Nevertheless, frequent calibration and high sensitivity to sensor error feedback pose challenges for this method. Cho et al. [55] revised this model to enhance the quality of measuring forces under low slip conditions. Their findings demonstrate that this methodology is more successful in situations where the classic high-slip value models are likely to fail. On the other hand, however, consistency in low friction conditions is a problem, and further model improvements for this reason are still needed [56, 57]. The work of the authors was conducted in association with the data comprising the vehicle speed, wheel angular position, and the applied wheel torque. They use state-of-the-art differentiation and observation techniques by which key parameters are estimated: wheel speed and acceleration, longitudinal and vertical forces, and the TRFC. In Table 1, the vehicular dynamics models, measurement processes, and parameter identification methods included in the various papers are presented.

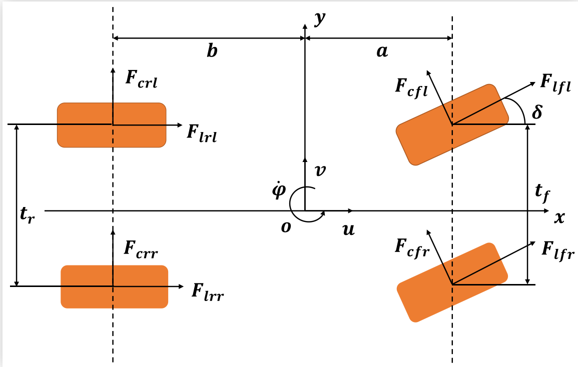
2.2.3. Lateral Dynamics Models Lateral dynamics methods generally involve less excitation than those using longitudinal methods. They are thus more applicable in estimating the TRFC when routine steering operations are happening. This property is needed, for example, in some systems such as lane keeping assist and stability control, which requires constant and minor correction. The components of TRFC estimation models that are based on lateral dynamics generally consist of lateral

TABLE 1 Summary of identification methods and vehicle dynamics model.

Vehicle model	Tire model	Methodology	Summary	References
Longitudinal vehicle model	Magic formula model	Kalman filter	1. High precision in high-slip rate scenarios; Easy access to sensor data; Low model complexity 2. Low slip rate scene accuracy is limited; Interference by the drive/brake system; Does not reflect lateral attachment characteristics	[51, 89]
	Magic formula model	Recursive least squares		[74, 75]
	Magic formula model	Reduced order observer		[40, 73]
	Magic formula model	Analytical model		[90]
	Magic formula model	Linear extended state observer		[91]
	Magic formula model	Improved nonlinear observer		[92]
	Magic formula model	Nonlinear curve fitting		[93]
	Magic formula model	Genetic algorithm		[94]
	Burckhardt model	Recursive least squares		[95]
	Burckhardt model	Nonlinear curve fitting		[21, 96]
	Modified Burckhardt model	Nonlinear estimator		[97]
	LuGre dynamic model	State observer		[48, 98]
	LuGre and Burckhardt models	T-S fuzzy and RLS		[99]
Lateral vehicle model	Brush model	Analytical model	1. Good adaptability to steering driving and high sensitivity to lateral force; Suitable for complex nonlinear scenarios; Multi-sensor fusion capability 2. High model complexity; Large disturbance by vehicle states; Limited accuracy at low speed	[100]
	Brush model	Nonlinear observer		[85]
	Hypothetical brush model	Direct model inversion		[101]
	Dugoff model	Unscented Kalman filter		[72]
	Modified dugoff model	Recursive least squares		[63]
	HSRI tire model	Dual extended Kalman filter		[79]
	TMeasy tire model	Nonlinear adaptive observer		[88]
	Magic formula model	Online gradient descent algorithm		[84]
Coupled vehicle model	Sakai's modified model	Unscented Kalman filter	1. Multi-dimensional data fusion improves identification accuracy; Enhance dynamic working condition adaptability; Reduced driving condition dependence 2. Model complexity and computational burden increase; The cumulative risk of sensor error increases; Parameter calibration is difficult	[102]
	Brush model	Analytical model		[103]
	HSRI tire model	Dual extended Kalman filter		[70]
	Load sensing bearing	Recursive least squares		[69]
	Neural network	Multilayer perception neural network		[104]
	Dugoff model	Limited-memory adaptive extended Kalman filter		[105]
	Magic formula model	Improved strong tracking unscented Kalman filter		[107]

acceleration, yaw angle rate sensors, and tire force models. All these will contribute gradually to an estimate of the forces that are being produced during steering maneuvers, which shows exactly the amount of friction there is between the tires and the road. As the driver makes steering inputs, sensors such as gyroscopes, accelerometers, and wheel speed sensors include data about key variables, among which are yaw angular rate, side acceleration, and wheel slip angle. This data is key to developing models that can efficiently register the behavior of the vehicle when it is in lateral motion. Algorithms such as the extended Kalman filter (EKF), UKF, and sliding mode observer (SMO) process sensor information to determine lateral tire forces. They are meant to reduce the noise and deal with nonlinearities in the dynamics of the vehicle. The tire model is linked with the measured lateral forces to predict the TRFC to a high level of accuracy. Four-wheel vehicle models are frequently used for this type of estimation modeling due to their suitability for comprehensively describing vehicle handling. The given schematics involve not only lateral motion but also spinning dynamics. Nonetheless, the complexity of the schemes presents challenges in real-time applications resulting from the high computational requirements. Figure 6 represents a model. The employment of the lateral dynamics model helps gain a more precise image of what is going on and properly handles all driving conditions. This is especially the case when the immediate

FIGURE 6 Dual-track 2-DOF vehicle dynamics model.



estimation of the TRFC is concerned, particularly when steering maneuvers are performed by the vehicle. This manner is highly applicable in driving conditions that do not require a big amount of "excitation." It is suitable for usual steering corrections and adjusting and some other low excitations as well.

The motion equations for lateral and yaw directions play a pivotal role in relating various state parameters that can be measured to this particular interaction force between the tires and the road surfaces. Thus, the equations form the framework for quantifying the transverse forces, which are mechanisms for identifying the TRFC accurately. The vehicle's kinematic behavior, the determinant of the vehicle's motion on the pavement, such as lateral accelerations and yaw angular velocities, can be related to the tire–pavement forces through the use of these equations. It helps to get a clearer grasp of the friction phenomenology later useful for designing structures of the contact between tire and road surface [58]:

$$\begin{aligned} m\left(\dot{v}+u\dot{\varphi}\right) &= F_{y_{fl}}+F_{y_{fr}}+F_{y_{rl}}+F_{y_{rr}} \\ I\ddot{\varphi} &= a\left(F_{y_{fl}}+F_{y_{fr}}\right)-b\left(F_{y_{rl}}+F_{y_{rr}}\right) \end{aligned} \tag{7}$$

where u is longitudinal velocity, v is lateral velocity, φ is yaw angle, a,b is the gravity center to the front and rear axle, m is vehicle mass, I is the moment of inertial around the z-axis, and $F_{yf,\,l}$, $F_{yf,\,r}$, $F_{yr,\,l}$, $F_{yr,\,r}$ are lateral forces on four tires.

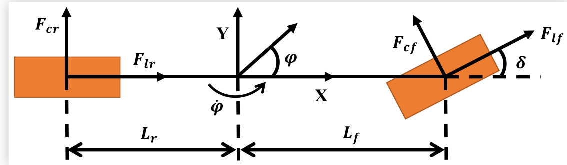
Tire forces are represented by:

$$\begin{aligned} F_{y_{fl}} &= F_{l_{fl}}\sin\left(\delta_f\right)+F_{c_{fl}}\cos\left(\delta_f\right) \\ F_{y_{fr}} &= F_{l_{fl}}\sin\left(\delta_f\right)+F_{c_{fl}}\cos\left(\delta_f\right) \\ F_{y_{rl}} &= F_{c_{rl}} \\ F_{y_{rr}} &= F_{c_{rr}} \end{aligned}$$

where F_{lf}, F_{lr} are longitudinal tire force of front and rear tires, F_{cf}, F_{cr} are lateral tire force of front and rear tires, δ is the front-wheel angle.

The four-wheel model can be simplified into the two-wheel, or bicycle model, by integrating the dynamics of the left and right wheels. This simplification reduces computational complexity while retaining essential dynamics, making it suitable for real-time TRFC estimation in control systems [59]. The bicycle model is widely used to describe the handling dynamics [60, 61], as shown in Figure 7.

FIGURE 7 Single-track 2-DOF vehicle dynamics model.



The simplified equations are written as:

$$\begin{aligned} m\left(\dot{v}+\dot{\varphi}u\right) &= F_{Y1}+F_{Y2} \\ I_z\ddot{\varphi} &= aF_{Y1}-bF_{Y2} \end{aligned} \tag{8}$$

The tire force on Y-axis can be expressed as:

$$\begin{aligned} F_{Y1} &= F_{lf}\sin\delta_f+F_{cf}\cos\delta_f \\ F_{Y2} &= F_{lr}\sin\delta_r+F_{cr}\cos\delta_r \end{aligned}$$

Lateral dynamics models link lateral and yaw motions with tire forces by integrating sensor data such as yaw rate and lateral acceleration. These models face challenges, including managing nonlinearities and sensor noise, but they are essential for accurate TRFC estimation in steering and cornering scenarios, and then based on the tire model mentioned above, the link between the whole vehicle state and tire slip angle and TRFC can be established [62, 63]. Lateral acceleration and yaw rate, which are readily measured by sensors such as accelerometers and gyroscopes, provide critical insights into vehicle handling dynamics. These measurements enhance TRFC estimation accuracy by directly reflecting the vehicle's response to steering inputs. To enhance TRFC estimation, sensor fusion techniques combining GPS, wheel speed sensors, and onboard measurements have been increasingly utilized. Methods such as Kalman filtering and Bayesian fusion algorithms integrate multiple data sources, improving estimation robustness and accuracy under varying conditions [64–66]. Table 1 reviews the lateral vehicle dynamics models, sensor measurements, and identification methods used in some studies.

2.2.4. Coupled Dynamics Models Longitudinal and lateral motions are inherently coupled during driving, and neglecting this coupling can result in significant underestimation of TRFC, especially during complex maneuvers like cornering or combined braking and steering. Coupled dynamics models extend traditional vehicle models by incorporating interactions between longitudinal, lateral, and yaw motions, providing a more comprehensive representation of vehicle behavior during complex driving scenarios:

$$\begin{aligned} m\left(\dot{u}-v\dot{\varphi}\right) &= F_{x_{fl}}+F_{x_{fr}}+F_{x_{rl}}+F_{x_{rr}} \\ m\left(\dot{v}+u\dot{\varphi}\right) &= F_{y_{fl}}+F_{y_{fr}}+F_{y_{rl}}+F_{y_{rr}} \\ I\ddot{\varphi} &= a\left(F_{y_{fl}}+F_{y_{fr}}\right)-b\left(F_{y_{rl}}+F_{y_{rr}}\right) \end{aligned} \tag{9}$$

where $F_{xf,\,l}$, $F_{xf,\,r}$, $F_{xr,\,l}$, $F_{xr,\,r}$ are longitudinal forces on four tires. Tire forces are represented by:

$$\begin{aligned} F_{x_{fl}} &= F_{l_{fl}}\cos\left(\delta_f\right)-F_{c_{fl}}\sin\left(\delta_f\right) \\ F_{x_{fr}} &= F_{l_{fr}}\cos\left(\delta_f\right)-F_{c_{fr}}\sin\left(\delta_f\right) \\ F_{x_{rl}} &= F_{l_{rl}} \\ F_{x_{rr}} &= F_{l_{rr}} \end{aligned}$$

Similarly, the bicycle model considering longitudinal, lateral, and yaw motion can be written as:

$$\begin{aligned} m\left(\dot{u}-\dot{\varphi}v\right) &= F_{X1}+F_{X2} \\ m\left(\dot{v}+\dot{\varphi}u\right) &= F_{Y1}+F_{Y2} \\ I_z\ddot{\varphi} &= aF_{Y1}-bF_{Y2} \end{aligned} \tag{10}$$

The tire force on X-axis can be expressed as:

$$\begin{aligned} F_{X1} &= F_{lf}\cos\delta_f-F_{cf}\sin\delta_f \\ F_{X2} &= F_{lr}\cos\delta_f-F_{cr}\sin\delta_f \end{aligned}$$

Identifying TRFC in lateral–longitudinal coupled states is challenging due to the intertwined effects of longitudinal and lateral forces on sensor measurements. Advanced sensor fusion techniques and adaptive filtering methods are being explored to address these complexities and improve estimation accuracy, which also places high demands on the tire model. Simultaneous estimation of tire longitudinal and lateral forces enhances TRFC accuracy by capturing the complete tire–road interaction. Techniques such as dual extended Kalman filters and recursive least squares algorithms are commonly used for this purpose [67–69]. Transverse–longitudinal coupling models require additional sensor inputs, such as GPS and LiDAR, to capture complex interactions between longitudinal and lateral dynamics. Recent methodologies use adaptive filtering and machine learning techniques to manage nonlinearities, enhancing model robustness and accuracy [70, 71]. Table 1 provides an overview of the vehicle dynamics models, sensor measurements, and identification methodologies used in several studies.

2.2.5. Identification Methods TRFC identification using vehicle dynamic models involves establishing relationships between measurable vehicle dynamics variables (e.g., lateral acceleration, yaw rate) and tire forces within a tire model framework. TRFC is treated as a variable to be identified, enabling dynamic adjustment of vehicle control systems based on real-time conditions. The core approach involves using regression algorithms to align measured data with model predictions, enhancing the estimation accuracy of TRFC. Algorithms such as Kalman filtering, recursive least squares (RLS), and SMOs are commonly used. Current regression-based TRFC identification methods include linear, extended, unscented, and cubature Kalman filters [51, 72, 73], which are effective in managing noise in dynamic systems. RLS [74, 75] and SMOs [76, 77] are also popular, each offering unique strengths in handling parameter estimation under varying conditions. The least squares method is often used for TRFC identification due to its simplicity and ease of implementation. However, the real-time performance and high data precision required by traditional methods pose challenges, particularly in complex or noisy environments, limiting their applicability [67]. To address this problem, RLS with a forgetting factor has been introduced to enhance adaptation to changing conditions by assigning more weights to recent data. Despite its advantages, this method still relies heavily on a large amount of high-precision data and has limitations in real-time performance, which makes it less suitable for application scenarios involving fast dynamic or noisy measurements [78]. To overcome these limitations, a second-order nonlinear extended state observer was developed to allow for faster and more accurate TRFC estimation. The observer uses a recursive formulation based on a simplified tire model, which improves the estimation speed and efficiently handles noise, which is particularly important in dynamic and variable driving conditions. However, as the complexity of the vehicle dynamics model increases, the RLS approach becomes difficult to handle nonlinear regression and is more sensitive to sensor noise, which can significantly degrade the accuracy of the estimation.

These challenges have led to the development of more advanced filtering techniques. Kalman filters, in particular, offer significant advantages over RLS methods. They effectively deal with measurement noise by using recursive estimation techniques and are able to adapt to variations in sensor inputs, which makes them more effective under real-world driving conditions. Thus, Kalman filters are very useful for TRFC estimation. As a specialized form of RLS, Kalman filters continuously update estimates based on sensor data, thus improving the accuracy and reliability of estimates in dynamic environments. The Kalman filter's continuous updating of sensor data-based estimates reduces the effect of noise, especially in noisy or rapidly changing conditions, making the estimates more reliable. Because sensor noise and the nonlinear behavior of vehicle dynamics affect estimation, a double-EKF method was introduced in Reference [79]. The unscented Kalman filter (UKF) avoids the need to calculate the Jacobian matrix, which makes it easier to use. It provides more accurate TRFC estimates than the regular EKF. The UKF's ability to manage nonlinear behavior makes it particularly useful for real-time vehicle control applications, where high precision is very important [80]. The sigma points generated by the cubature Kalman filter (CKF) are more stable in the high-dimensional state, and the annual dimension of computational complexity increases more slowly. Gao et al. proposed a TRFC identification method based on double-square-root volumetric Kalman filtering (DSRCKF), which is superior to UKF and CKF [81]. Nevertheless, these Kalman-based approaches are restricted to Gaussian distributed noise. Particle filters are highly effective for non-Gaussian and nonlinear systems due to their flexibility in modeling complex probability distributions. However, they can be computationally intensive, making their real-time application challenging in resource-constrained environments. Liu et al. [82] introduced a predictive approach combining an auxiliary particle filter with an iterative estimator, enhancing TRFC estimation accuracy and robustness. Real vehicle tests demonstrated the method's effectiveness, particularly in handling abrupt changes in road conditions. State observer methods, including the extended Luenberger observer [83], online gradient descent algorithm [84], nonlinear observer [85], and high-order SMO [86], offer robust TRFC estimation by continuously adjusting to system dynamics. However, the observer design and tuning parameters often limit their performance, necessitating adaptive techniques for broader applicability. The observer-based approach typically has a defined range of applicability, prompting the proposal of an adaptive observer [87, 88] designed to address this limitation across all road conditions. Table 1 presents estimation methods grounded in the vehicle dynamics model to facilitate a clearer comparison of the various methodologies.

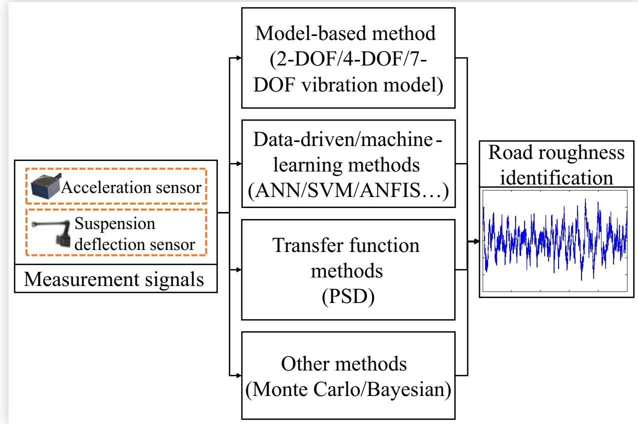
3. Road Roughness Profile

Vehicle vibrations are primarily excited by the road surface profile, making its identification crucial for effective control of vehicle vertical dynamics. Accurate road profile estimation directly influences suspension performance, ride comfort, and overall vehicle safety. As shown in Figure 8, road profile estimation methods include model-based or observer methods that rely on physical models, data-driven and machine learning techniques that utilize pattern recognition, and frequency response/transfer function methods that analyze vibration data. Each method offers distinct advantages, such as real-time applicability or high accuracy, but also faces specific limitations such as computational demands or sensitivity to noise. These are described in the following sections.

3.1. Vehicle Vertical Vibration Model

The quarter vehicle model, a basic 2-DOF vibration model, captures the essential dynamics of vertical vehicle motion and serves as a foundational tool in road profile estimation. Its simplicity allows for quick analysis, although more complex models, such as half-vehicle or full-vehicle models, are often used to capture additional dynamics. This model is shown in Figure 9.

FIGURE 8 Road roughness profile estimation framework.

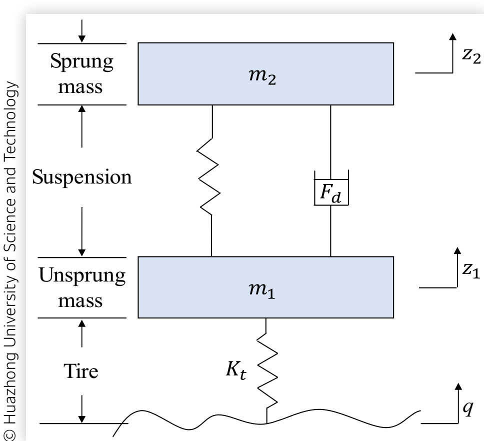


The equations are written as:

$$\begin{cases} m_2\ddot{z}_2 + C\left(\dot{z}_2 - \dot{z}_1\right) + K\left(z_2 - z_1\right) = 0 \\ m_1\ddot{z}_1 + C\left(\dot{z}_1 - \dot{z}_2\right) + K\left(z_1 - z_2\right) + K_t\left(z_1 - q\right) = 0 \end{cases} \tag{11}$$

where m_1 is unsprung mass; m_2 is sprung mass; K is air spring stiffness; C is absorber damping; K_t is tire stiffness; q is road excitation input; z_1 is tire vertical displacement; z_2 is vehicle body vertical displacement; Building on the quarter vehicle model, more complex models such as the 4-DOF half-vehicle and 7-DOF full-vehicle models have been developed to capture a broader range of dynamics. These advanced methods that account for additional factors, such as anti-roll and others, bring in high accuracy of estimation. Therefore, they are particularly good for active suspension control and road profile estimation.

FIGURE 9 2-DOF vibration model.



3.2. Model-Based Observer Methods

Kalman filters (KFs) and SMOs are often used for the estimation of road profiles due to their robustness in dealing with noise and adapting to dynamic conditions. Recent advances have introduced adaptive KF and higher-order sliding mode observers, which significantly improve the accuracy and perform well even in rapidly changing road condition environments. Depending on the situation, different types of KFs are applied. The linear KF (LKF) is suitable for simple linear systems, while the EKF is designed to accommodate nonlinear relationships common in vehicle dynamics. The uncsented KF (UKF) is particularly advantageous in strongly nonlinear scenarios because it does not require complex Jacobi matrix calculations. These variants are applied to various aspects of road profile estimation, depending on the system's complexity. In 2011, the linear quarter-car model became the foundation for many KF applications [105]. This model uses data such as suspension deflection, body position, and acceleration. However, its simplicity can limit accuracy, particularly in highly dynamic or nonlinear scenarios. To address these limitations, more advanced models have been developed, incorporating additional vehicle parameters and adaptive filtering techniques to enhance estimation precision. Later improvements followed. One enhanced KF accounted for changes in the vehicle's sprung mass [106]. Another augmented KF made use of all available sensors [107]. A modified KF framework [108] was introduced for identifying parameters. It helped improve the localization of autonomous vehicles within nonlinear spring-damper systems. While these adaptations enhanced accuracy, they still face challenges in scenarios with high noise levels or rapidly changing road conditions. One of the persistent challenges with KF methods is tuning the covariance matrix. This process is often performed through trial and error, which can result in suboptimal performance. To address this, recent advancements have focused on adaptive covariance tuning techniques. In addition, machine learning algorithms are increasingly being applied to dynamically adjust filter parameters to improve estimation accuracy. Researchers have proposed several innovative solutions. For example, an algebraic estimator [109] updates the covariance matrix based on changes in road roughness [110]. Another approach combines an adaptive KF with an adaptive super-helical observer (AKF-ASTO) algorithm [111] to further improve the accuracy under different road conditions.

SMOs are highly powerful instruments for the assessment of pavement profiles, mainly when the road profile is treated as a constant unknown. These observers are highly resistant to both external sources of error and internal calculation errors, which leads to effective assessment of dynamic environments. Still, they are more prone to signal interference and require fine adjustments to achieve excellent results. Hence, the first approach is to use a full-vehicle model with 16 degrees of freedom, which is supposed to replicate intricate movements of the vehicle in the vertical, roll, and pitch planes. The model helped to explain how the road surface profile affects the overall stability and control of the vehicle [112]. The findings of the research showed how the pavement affects the overall stability and control of the vehicle. This variable was addressed to a larger extent in a second-order sliding mode observer over time. Shaping the observer increased the elasticity of the model to react to different driving speeds and road conditions [113]. The example of minimizing these dynamic interactions became a major opportunity for improving road profile accuracy estimation. New approaches of higher-order SMOs and adaptive super-helix observers are addressing this problem now [114]. They significantly increase the complexity and dynamics of driving scenarios by introducing conditions that are outside of the control of observers. Therefore, their consistent and frequent implementation is necessary. These advanced methods offer an alternative to parameter identification and tire friction estimation when the standard observers no longer work well [115, 116]. Consequently, the movement control of automobiles in different road conditions is improving more and more. As a result of the SMO in combination with an adaptive KF, it has been found that the dynamic properties of tires in active suspension control can be adapted more appropriately to varying road conditions [117]. Other approaches, such as adaptive observers and Q-parameterization methods [118] are aimed at dynamically adjusting the parameters of observers to reduce the computational cost while preserving good performance. Blindfolding is scaffolding the aforementioned methods and allowing experimental to prove good results in real-time pavement profiling [119, 120]. The beliefs of the Q-parameterization approach are confirmed. It outperformed KFs, especially due to its lower computational needs and easy implementation. However, using the presented approach makes it possible to estimate and control real-time pavement profile.

The state observer in [121] uses the dynamic response of preceding vehicles to generate preview control inputs for follower vehicles, enhancing control accuracy and coordination in platoon driving scenarios by anticipating vehicle movements and improving inter-vehicle spacing management. The H∞ observer is feasible for real-time applications, particularly in active suspension control, due to its robustness in handling system uncertainties. However, it requires extensive knowledge of vehicle parameters, which limits its practical use in scenarios with incomplete data [122]. The jump-diffusion process estimator helps detect and identify parameters at the same time. It provides strong and reliable control, even when road conditions change suddenly. This makes it useful in situations with abrupt changes in the road surface. However, the estimator's performance can be affected by its complex modeling needs. It also requires a lot of computational power, which can limit its effectiveness in some cases [123]. Feng et al [124] used the ratio of sprung acceleration square to vehicle speed as an evaluation index of road roughness, which plays a positive role in the mode switching of semi-active suspension. Although these estimators are effective in active suspension control, they face significant challenges, including extensive modeling requirements that demand high computational resources and sensitivity to speed variations, which can compromise estimation accuracy during rapid vehicle maneuvers. Recent advancements include new techniques for adaptive modeling and real-time parameter adjustments. These improvements use methods such as adaptive observers based on machine learning and algorithms that adjust settings online. These new methods increase the reliability of the estimators. They also make the estimators more useful in different driving situations.

3.3. Data-Driven/Machine Learning Methods

Machine learning (ML) methods have enabled the creation of many algorithms to measure and identify road surfaces. Some of the most commonly used techniques include neural networks (NN), artificial neural networks (ANN), and support vector machines (SVM). Among these, NN/ANN and SVM are the most popular for road condition analysis. In 2010, a study [125] introduced a Bayesian regularized nonlinear autoregressive exogenous model (NARX) to evaluate pavement roughness (PR). This model used acceleration data collected from a linear half-vehicle system. ANN-based methods have shown great success in identifying road surfaces across different environments. These environments include both specialized applications, such as mining vehicles and rough terrain [126], as well as standard passenger vehicles such as the Land Rover Defender 110 [127]. This success is due to ANN's ability to adapt to a variety of input conditions. A similar approach is described in [128], where seven different vehicle acceleration variables were used as inputs for an ANN model. Combining ANN with wavelet analysis has further improved estimation efficiency. Wavelet analysis breaks down complex signals, making the system more effective, especially in connected vehicle networks [8]. In [129], ANN was also used with a ratio. This ratio involved dividing the mean square value of unsprung mass acceleration by the vehicle's speed to classify road power spectral density (PSD). This method performs well, regardless of changes in vehicle speed or suspension characteristics, ensuring consistent road condition classification.

Different methods have been used for the terrain type classification. Among these, ANN, BP, SVM, PCA-based algorithms, and even self-organizing maps are beneficial in this regard [130–133]. They decrease the size of the data and highlight the most important characteristics. Thereby, the classification accuracy is improved [134, 135]. There is a scope for using the hybrid approaches of SVM and wavelet analysis or FFT to determine the impact of variation of speed on the process of terrain classification. This integration improves the model's robustness and eliminates the negative effect of speed-induced noise [136, 137]. In terms of more novel approaches, besides the traditional methods of ANN and SVM, more advanced ML algorithms (extensive DNN and the adaptive neuro-fuzzy inference system (ANFIS)) have been the subject of some researchers' studies [138]. The key advantage of these tactics is their potency in nonlinear systems with extremely complicated dynamics. An example of such a system is data obtained with a gas chromatograph [139]. Deep NN and probabilistic neural network classifiers were included in the work because it is possible to use the measured system responses [140]. ANFIS for road classification using wavelet analysis based on spring-mass acceleration [141] demonstrated superior performance compared to other methods [142]. ANFIS was enhanced with KF to improve real-time adaptability and estimation accuracy in semi-active suspension systems, demonstrating superior control performance in varying road conditions [110, 143]. Additionally, the probabilistic neural network (PNN) classifier, employing wavelet analysis, outperformed ANFIS and NARX methods. Integration with adaptive Kalman filtering with adaptive set theoretic observers (AKF-ASTO) [111] facilitated adaptive adjustments of process noise covariances Q and R for the KF, resulting in enhanced accuracy. The random forest (RF) classifier effectively fuses time and frequency domain data, enhancing classification accuracy for controllable suspension systems. When combined with transfer functions, RF enables speed-independent road classification, significantly improving its applicability in real-world driving conditions [144]. Independent component analysis has been recognized as a simple, rapid approach for feature extraction, while comparative studies of ML techniques highlight the trade-offs between complexity, speed, and accuracy, guiding the selection of appropriate methods based on specific application requirements [145, 146].

3.4. Transfer Function Methods

The transfer function method was proposed by Gonzalez [147] to estimate road PSD based on the relationship between the road surface and vehicle acceleration:

$$H(\Omega) = \frac{PSD_{acc}(\Omega)}{PSD_{road}(\Omega)} \tag{12}$$

where $PSD_{acc}(\Omega)$ and $PSD_{road}(\Omega)$ are the PSD of vehicle acceleration and road profile.

The road can be classified according to ISO 8608 [51] based on estimates from PSD_{acc} of the axle or body acceleration measurements [147]. Numerous methods have been developed for road profile estimation, including the transfer function (TF) approach, which has proven efficient in modeling the relationship between vehicle acceleration and road roughness (Table 2) [148]. The TF technique was further extended to full-vehicle models, enabling the estimation of road PSD across varying speeds, enhancing the method's applicability in real-world scenarios [149]. Dynamic tire pressure sensors have been used to estimate road profiles by assuming a linear relationship between tire pressure and road surface irregularities within the TF framework, offering a novel approach to road condition monitoring [150].

Alternative methods, such as numerical optimization using Monte Carlo simulations, have been employed to derive optimal pavement response, although these methods often involve high computational costs [151]. The control constraints method focuses on tire dynamics and requires solving complex differential algebraic equations, offering precise modeling at the expense of increased computational complexity [56]. The modulating function technique is effective for real-time estimation and noise suppression, making it particularly useful in off-road vehicle applications where signal quality can be compromised [152]. Bayesian estimators, as proposed in [153], offer flexibility across different vehicle models but require accurate prior knowledge of road conditions to maintain estimation accuracy. Microphones have been suggested as alternative sensors for capturing tire noise, offering a novel approach to PR estimation. However, this method faces challenges such as signal contamination from environmental noise, necessitating further research to improve robustness [154].

4. Road Slope

While many studies focus on flat road surfaces, real-world driving conditions often involve varying slopes and cambers, significantly impacting vehicle dynamics and control systems. Compared to flat surfaces, the vertical component of gravitational acceleration on inclined surfaces is smaller, while there are additional horizontal components in both lateral and longitudinal directions. Inaccurate accounting for road pitch and roll can result in substantial errors in vehicle state estimation, affecting control accuracy and overall vehicle stability. Hashemi et al. [7] identified key challenges in road pitch and roll estimation, including unknown friction coefficients, difficulty isolating road slope effects, and the lack of affordable, accurate measurement tools, which collectively hinder reliable slope estimation. As shown in Figure 10, road slope estimation techniques are generally categorized into kinematic, dynamic, and hybrid approaches, each offering distinct advantages such as simplicity or accuracy but also facing limitations like noise sensitivity or computational complexity.

4.1. Model-Based Observer

Kinematic model-based methods treat the vehicle as a point mass and utilize kinematic observers to estimate slope without requiring detailed vehicle parameters, but these models are highly sensitive to noise, especially at low speeds [155, 156]. These methods do not require knowledge of vehicle parameters, tire models, or TRFC and can provide road slope estimates in many cases. However, they are sensitive to measurement noise, particularly at low longitudinal or lateral velocities, and disturbances from accelerometer measurements in both directions may further degrade estimation performance.

Vehicle dynamics models used to estimate the slope of a roadway usually need to consider the dynamics of vehicle roll and pitch, taking the effect of road bank and grade into account in the equations. The roll motion is shown in Figure 11.

The equation of roll motion can be written as:

$$\left(I_x + m_s h_{RC}{}^2\right)\ddot{\varphi} = -K_\varphi \varphi - C_\varphi \dot{\varphi} + m_s h_{RC} \cdot u_\varphi \tag{13}$$

where I_x is the moment of inertia of roll axis; m_s is the sprung mass; h_{RC} is the roll center height; K_φ is the roll stiffness; C_φ is the roll damping; u_φ is the unknown input considering road bank:

$$u_\varphi = \dot{V}_y + rV_x + g\sin\left(\bar{\varphi}_v + \varphi_r\right)$$

where $\bar{\varphi}_v$ is the measured roll angle; φ_r is the road grade; V_x, V_y are longitudinal and lateral velocity; r is the yaw rate.

The pitch motion is shown in Figure 12.

The equation of pitch motion can be written as:

$$\left(I_y + m_s h_{PC}{}^2\right)\ddot{\theta} = -K_\theta \theta - C_\theta \dot{\theta} + m_s h_{PC} \cdot u_\theta \tag{14}$$

where I_y is the moment of inertia of roll axis; m_s is the sprung mass; h_{PC} is the roll center height; K_θ is the roll stiffness; C_θ is the roll damping; u_θ is the unknown input considering road bank:

$$u_\theta = -\dot{V}_x + rV_y + g\sin\left(\bar{\theta}_v + \theta_r\right)$$

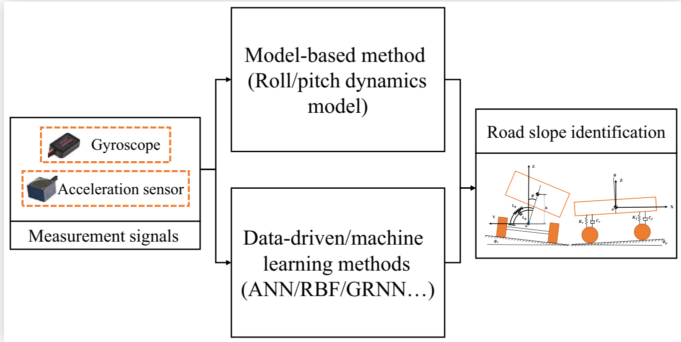
where $\bar{\theta}_v$ is the measured roll angle; θ_r is the road bank; V_x, V_y are longitudinal and lateral velocity; r is the yaw rate.

Dynamic model-based approaches use advanced state observers, such as KF, to incorporate vehicle mass and inertia, resulting in more robust slope estimation compared to simpler kinematic models [157]. Compared to kinematic models, dynamic models account for forces acting on mass and inertia, providing relatively robust vehicle state predictions. SMOs adaptively estimate road slope using bicycle and tire models, offering enhanced accuracy in nonlinear conditions and proving particularly useful in dynamically changing environments [158, 159]. Coupling road slope estimation with other vehicle

TABLE 2 Summary of road roughness identification methods.

Methods	Algorithms	Summary	References
Model-based observer	Kalman filters	1. Real-time processing of sensor input, in line with low latency requirements; Fusion of multi-source data can provide stable estimation results when there is interference. Combined with some kinds of KFs, the nonlinear factors of the estimation can be considered. 2. Complex models may increase the computational complexity and increase the computational time; Algorithm performance is highly dependent on system model parameters.	[105–108, 111]
	Sliding mode observers	1. Strong anti-interference ability and parameter robustness; Reduce the dependence on external sensors; Suitable for nonlinear systems 2. High-frequency switching control characteristics will produce chattering; It is sensitive to the accuracy of model parameters and requires a lot of work during calibration.	[112–116]
	Robust observers	1. Strong anti-interference ability to model uncertainty and external disturbance 2. High computational complexity.	[122]
Data driving/machine leaning	NN/ANN/DNN	1. Strong nonlinear modeling ability, better than a dynamic model under abrupt or extreme road conditions; Combined with vibration signal or visual texture analysis, the classification accuracy is high; In real applications, the prediction speed is fast and the real-time performance is strong 2. High data dependence and labeling cost; Model interpretability is poor; Lack of adaptability to dynamic environments.	[126–129]
	SVM	1. Small sample learning ability; The computational complexity is low, which can improve the computational efficiency and real-time performance. It has strong interpretability and is convenient for fault diagnosis and model optimization 2. Limited extraction of nonlinear characteristics; The ability of using multi-source data fusion is weak; Poor expansion when entering high-dimensional data.	[134–137]
	ANFIS	1. Combined with fuzzy rules, more explanatory; Can learn and adjust parameters 2. High complexity of parameter optimization; Low computational efficiency.	[110, 138, 141–143]
Transfer function	PSD	1. High computational efficiency; Suitable for stationary random processes; Low parameter dependence 2. High noise sensitivity; The adaptability of model parameters is poor; Non-gaussian features cannot be characterized.	[147–150]
Others	Monte Carlo	1. Strong modeling ability of complex systems; Strong global optimization ability, reducing uncertainty 2. High calculation cost and poor real-time performance; The result is weak in interpretation.	[151]
	Control constraints method	1. Enhance algorithm robustness; Optimize feasible solution space; Noise suppression 2. Increased computational complexity; High model dependence.	[152]
	Bayesian estimator	1. Quantify the uncertainty; Suitable for non-Gaussian noise environment; Have prior knowledge integration ability 2. High computational complexity; Prior distribution sensitivity.	[153]
	Modulating function technique	1. Strong noise robustness and anti-interference ability; It is suitable for the identification of unsteady systems 2. High computational complexity; Modulation function selection depends on experience; High requirement for prior knowledge.	[154]

FIGURE 10 Road slope identification framework.

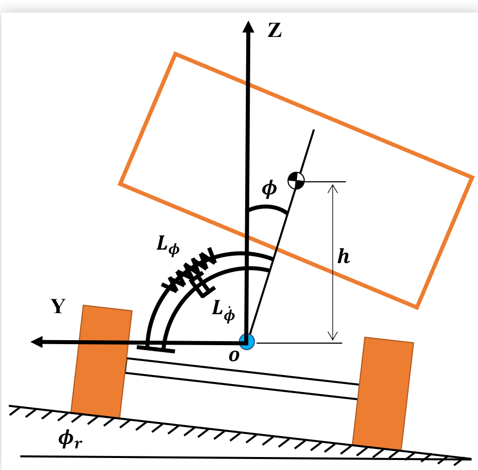


parameters, such as tire stiffness and friction coefficients, improves overall estimation accuracy but increases model complexity and data requirements [160]. Lyapunov-based tire lateral stiffness observers proposed in [161–163] can improve the accuracy of slope estimation but require continuous input excitation and exhibit reduced adaptivity outside the linear tire model region.

4.2. NN-Based Observer

Machine learning-based techniques have been proposed to augment traditional estimation methods in [164, 165]. However, performance gaps may arise when data availability is limited. Combining kinematic and dynamic estimation methods helps leverage the strengths of both, though it can lead to estimation discontinuities due to model inconsistencies, complicating real-time applications [166–168]. Reference [169] proposes using a kinematic model to observe tire cornering stiffness and then using a dynamic model to estimate road slope and sideslip angle. [7] propose a road angle estimation method independent of road adhesion coefficients, utilizing a vehicle kinematic model to estimate vehicle pitch and roll angles, studying the correlation between road angle and vehicle pitch/roll using a vehicle dynamic model, and simultaneously estimating road pitch and roll angles, thus improving the accuracy of road angle estimation.

FIGURE 11 Roll motion dynamic model.



5. Vehicle States' Estimation

For automotive electronic control functions to perform their intended roles, certain physical parameters related to the vehicle and its environment must be known to serve as the basis for decision-making. Accurate acquisition of these parameters is a prerequisite for the design and operation of all active safety systems. While some of these parameters can be directly measured by onboard sensors, critical vehicle states such as the sideslip angle, longitudinal vehicle speed, and roll angle are typically not directly measured due to technical limitations or cost considerations. Instead, these states are estimated using readily measurable quantities such as wheel speeds, steering wheel angle, lateral and longitudinal acceleration, and yaw rate, which can be obtained through low-cost sensors. Observations of parameters like the sideslip angle, roll angle, and tire forces are challenging to obtain directly from sensors. Consequently, constructing vehicle state observers has become the primary method for acquiring these vehicle states. Based on the literature, as

FIGURE 12 Pitch motion dynamic model.

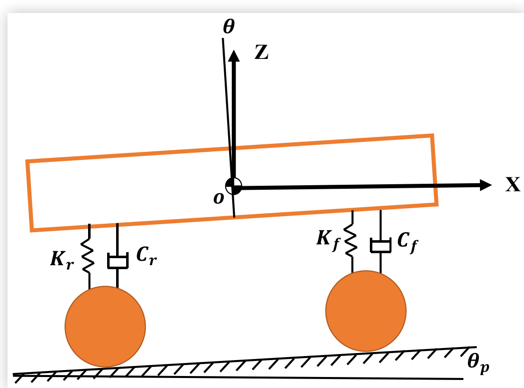
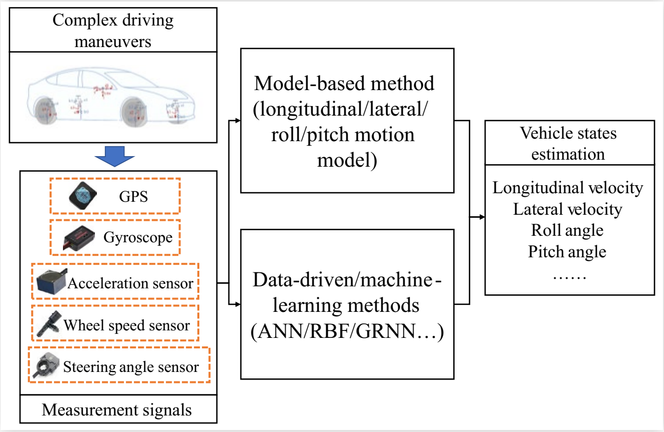


FIGURE 13 Vehicle states estimation framework.



shown in Figure 13, vehicle state estimation methods can generally be categorized into two main approaches: model-based observers and neural network-based observers.

5.1. Model-Based Observer

Similar to road condition identification, observers are based on vehicle models, which are generally classified into kinematic and dynamic models. Kinematic models typically consider only the vehicle's motion without accounting for forces. However, the main issue with estimation using kinematic models is that the system becomes unobservable when the vehicle's yaw rate is low. Dynamic models, as described in Section 2.2, take into account the forces acting within the system and require the integration of tire models to express longitudinal and lateral forces as functions of slip ratio and tire slip angle, respectively. While dynamic models provide better vehicle state estimation, the accuracy of the estimation largely depends on the precision of the tire model. To address this issue, various tire parameter updating algorithms have been proposed, such as adjustments to the coefficients of the magic formula and tire cornering stiffness.

In the literature, three primary types of state observers are frequently discussed: the Luenberger observer (LO), the SMO, and the KF along with its variants. Both LO and SMO are deterministic observers that can be used for linear as well as nonlinear systems. However, these methods assume that the system and its inputs are fully known and do not take into account possible modeling inaccuracies or noise in the measurements. On the other hand, KF-based observers are designed specifically to manage stochastic systems, making them well-suited for handling model uncertainties and measurement noise effectively [170]. In Reference [171], a comparison was made between the EKF and the SMO using a nonlinear heavy vehicle model. The results showed that both methods were accurate enough. However, the SMO was preferred because it needed fewer input measurements. The EKF is known for being simple, stable, and strong, while also being good at managing input and measurement noise. These qualities make it the most commonly used observer for estimating vehicle states [172]. This conclusion is further supported by [173], where the EKF, LO, and SMO were evaluated using a nonlinear single-track model for vehicle state estimation (VSE). The results showed that the EKF had a smaller estimation error compared to both the LO and SMO.

Several studies, such as [166, 174], suggest using the KF observer, usually an EKF, in a complex vehicle model with seven degrees of freedom (7-DOF). These models also include fully nonlinear tire models such as the Pacejka or dugoff models. As mentioned earlier, there is a need to find a balance between the complexity of the model, the accuracy of the results, and the computing power needed. In addition, when working with a dynamic model, getting all the parameters needed for tire modeling from manufacturers can be difficult. This is because this type of information is often considered private and confidential. As mentioned in Section 2.1, when actual conditions differ from the modeled conditions (e.g., due to tire wear), the accuracy of VSE decreases. To address this issue, some researchers [175, 176] have proposed a simplified single-track model incorporating adaptive cornering stiffness to mitigate the impact of modeling discrepancies. The results indicate that this method provides vehicle state estimates that are generally close to actual values. However, certain peak conditions still exhibit deviations, primarily because the tire model operates only within the linear region, neglecting the behavior in the nonlinear and saturation regions, which leads to increased estimation errors.

In a related study [177], the tire model was adjusted to fit the entire nonlinear range by modifying the Pacejka model dataset and applying a tuning process with the EKF observer. This allowed the model to match the lateral vehicle behavior. The tuning process involved changing a single, easily adjustable parameter, which made it possible to use a simpler vehicle model without needing detailed knowledge of the tire parameters. This approach also considered the nonlinear saturation behavior of tires, achieving an estimation error of less than 5% across all maneuvers tested. It demonstrated high computational efficiency, making it well-suited for real-time applications. Ahangarnejad et al. [178] proposed a dual-EKF (DEKF) system, utilizing two EKFs running in parallel. The first EKF estimates vehicle states such as sideslip angle; the second EKF adjusts the parameters of the rotating wheel model. The updated parameters are then fed back into the first EKF to refine the vehicle state estimates. Chen et al. [179] also used DEKF to estimate both the lateral speed and the sideslip angle, and update the tire parameters in real time. Wang et al. [180] designed a second-order fault-tolerance extended Kalman filtering (SOFTEKF) that can effectively reduce data loss and improve estimation accuracy. Naets et al. [181] developed a nonlinear least squares estimator for tire parameters. Although this estimator can be run online, it does not provide real-time performance.

One of the major drawbacks of the EKF is its reliance on Jacobian matrix calculations, which are computationally intensive and time-consuming. Additionally, EKF requires very short sampling periods to maintain estimation accuracy, further increasing its computational burden. To address these limitations and ensure the precision of VSE, researchers commonly adopt the UKF as an alternative to the conventional EKF. Recent research has increasingly focused on observer-based methods using the UKF due to its superior performance in VSE [165, 182–184]. The UKF is particularly effective in handling nonlinearities, making it a preferred alternative to the EKF. Unlike the EKF, which requires Jacobian matrix calculations, the UKF employs an approximation technique based on sigma points that capture the statistical behavior of state variables [185]. These sigma points serve as nonlinear representations of the system, allowing the UKF to describe system behavior more accurately. By incorporating these sigma points into an approximate linear space within the Kalman gain equation, the UKF eliminates the need for Jacobian computations, making it more feasible for systems with significant nonlinearities. However, while UKF provides more accurate final results than EKF, it also introduces additional complexity. The placement of sigma points must be carefully determined, and their weights must be meticulously adjusted to ensure an accurate representation of the vehicle's state space. Antonov et al. [186] emphasized that selecting appropriate sigma points is crucial for achieving precise state estimation. Despite these complexities, the UKF offers a more realistic and accurate representation of frequency fluctuations, which is particularly beneficial for flexible and highly dynamic systems.

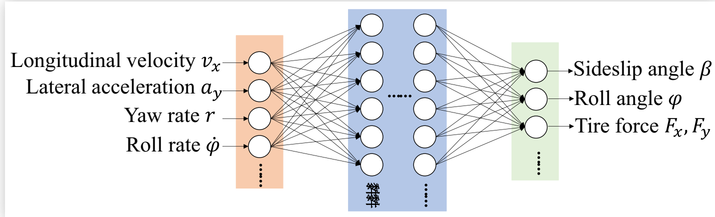
In [182], Chen et al. elaborated a new version of the UKF by implementing an adaptive VS observer with dynamic corrections. This solution makes it possible to overcome the problem of model uncertainty with the help of adaptive parameters, and the sensor-based observer corrects the nonlinear region error and increases the model reliability in intricate situations. Li et al. [61] updated the conventional KF with the new KF variant. To make this modification possible, this devised filter hybrid incorporates two KF processes: Square-root cubature KF (SCKF) and square-root cubature-based receding horizon Kalman filter (SCRHKF). The SCKF works with multi-dimensional integrals that are efficiently solved with high accuracy and a fast convergence rate using the order of spherical radial cubature rule; however, it makes them more vulnerable to uncertainty and noise [187]. On the other hand, the SCRHKF is more capable of dealing with uncertainty and noise; however, it leads to a reduction in the speed of convergence, as it is based on a smaller number of measurements. In this way, while synthesizing these two filters: SCKF and SCRHKF, they are providing a robust VSE strategy. Li and Zhang avoided a system's shortcomings by developing a hybrid KF and state estimation system. Li and Zhang [61] further developed a hybrid KF for VSE, which fully utilizes the best features of the SCKF and the SCRHKF while addressing their respective limitations. The results showed low RMS errors and small peak errors in steady-state maneuvers, which verified the accuracy of the method. However, the strength of the method is its intensive computational properties, which makes it difficult to apply them in real situations. The improvements are still important in order to develop its functionality further and to make it part of practical applications.

In summary, VSE relies on both kinematic and dynamic models, with dynamic models offering higher accuracy but being heavily dependent on tire model precision. KF-based observers, particularly EKF and UKF, are widely used due to their ability to handle model uncertainties and measurement noise. While EKF is commonly applied, it has limitations related to Jacobian matrix computation, leading to the adoption of UKF for improved handling of nonlinearities. Recent research has explored adaptive and hybrid KF approaches, such as DEKF, SCRHKF, and hybrid SCKF-SCRHKF, to enhance estimation accuracy and robustness. However, computational complexity remains a key challenge for real-time applications.

5.2. NN-Based Observer

There are only a limited number of studies in the literature focusing on VSE using ANNs, as shown in Figure 14. This approach is specifically designed to eliminate the need for complex vehicle models and extensive parameter sets, particularly those related to tire dynamics. Additionally, using ANNs minimizes errors associated with signal integration in noisy environments. ANNs are capable of estimating the vehicle states by capturing intricate relationships between input and output data using relatively

FIGURE 14 ANN diagram.



simple mathematical operations. A comprehensive mathematical framework for ANNs is provided. For clarity, Figure 14 illustrates a high-level schematic of the ANN-based estimation process.

For an ANN to make accurate estimations, a well-prepared training dataset is crucial [188, 189]. Most studies use the Levenberg–Marquardt backpropagation (BP) algorithm along with a mean squared error (MSE) performance index to train the ANN. The exception is Yu et al. [187], which uses a radial basis function (RBF) method. The BP optimization technique updates the weights and biases of the network through an iterative process, as described by Levenberg and Marquardt. On the other hand, the RBF approach is a feed-forward method that requires less computation and reaches convergence more quickly [190].

Recent research has explored the use of general regression NN (GRNN), a variant of RBF networks known for their effective learning and high function approximation capabilities [191, 192]. The main difference between a GRNN and a standard RBF network is the addition of a special linear layer. This layer calculates a weighted sum of the outputs from the first layer. Wei et al. [193] showed how this type of ANN could be used in different vehicle handling scenarios, achieving quick responses and accurate estimations. However, they did not evaluate how well this method worked when conditions changed, such as at different speeds or with varying road grip. Some studies suggest other strategies to improve the efficiency and reliability of ANN-based estimation. For instance, Chindamo et al. [194] suggested a specific training maneuver to optimize the ANN's estimation performance while maintaining a simple network structure to reduce computational complexity, using only 10 neurons in the hidden layer. The training maneuver involved a 45° single steer test with speeds increasing from 5 km/h to 100 km/h at a rate of 15 km/h per min, conducted under three distinct friction conditions (i.e., μ = 1, 0.5, 0.2). Although this approach yielded promising estimation results in simulations using the CarSim platform, real-world validation poses challenges due to the extensive proving ground required for such testing. Marotta et al. [195] used multi-output NN to estimate the longitudinal, lateral, and vertical forces of tires simultaneously, which is of great benefit to the identification of TRFC.

In a significant study, Broderick et al. [196] trained an ANN using different maneuvers to account for changes in vehicle weight, road conditions, and tire properties. Although this approach was thorough, the training process took a lot of time and was tested only under two specific situations. Similarly, several researchers, including Acosta et al., Alagappan et al., and Huang et al. [197–199], used hybrid estimators that combine a neural network with an observer, typically an EKF. In these methods, the neural network is used only for fitting the tire data, while the observer estimates the vehicle dynamics states. To keep computational complexity low, a simpler neural network structure was used in these studies. The main limitations of ANN-based estimation are its sensitivity to changes in vehicle dynamics after training and the difficulty in estimating road banking angles without extra algorithms. Additionally, as mentioned in [200], filtering out the part of lateral acceleration caused by gravity is necessary to improve estimation accuracy (Table 3).

6. Summary and Perspectives

In this article, a comprehensive review and comparative analysis of classical road condition identification and VSE algorithms are presented, focusing on key factors such as TRFC, road roughness profile, road slope, and grade angle. Existing studies are mainly classified into experiment-based, model-based, and AI-based approaches, where experiment-based approaches directly use sensor data to estimate target parameters, relying on the acquisition and analysis of a large amount of measured data; model-based approaches require the establishment of a mathematical model between the sensor data and the estimated target to describe the physical characteristics of the system, and then use estimation algorithms to approximate to obtain the estimation results; AI methods use techniques such as NN to directly establish mapping relationships between sensor data and estimated targets, omitting the explicit modeling process. At present, most studies have been optimized and expanded around these three types of methods, forming a more mature theoretical framework. However, despite the significant progress made in various fields, and the fact that the road state and vehicle state are inextricably linked and interact with each other in practical engineering applications, it is still difficult for existing studies to comprehensively solve the practical problems due to the complexity of road

TABLE 3 Summary of vehicle states estimation methods.

Methods	Algorithms	Summary	References
Model-based observer	Kalman filters	1. Multi-source information fusion and optimal estimation; High real-time and computational efficiency 2. Strong dependence of noise statistical characteristics; Model error sensitive; The nonlinear modified Kalman filter increases the computational complexity in high-dimensional space	[165, 166, 170, 171, 174–180, 182–187]
	Sliding mode observers	1. Strong anti-interference ability; High computational efficiency; Fast convergence 2. Buffeting problem; The estimation accuracy is low at low speed. Parameter adjustment is complicated; Strong model dependence	[171]
	Luenberger observer	1. Simple structure and efficient calculation; The convergence speed is adjustable 2. Limited anti-interference ability; Parameter adjustment depends on experience; Lack of explicit noise processing mechanism	[173]
Data driving/ machine leaning	ANN	1. Strong nonlinear modeling ability; Strong anti-noise and outlier ability; Multi-source information fusion has great potential 2. High data requirements and training costs; Difficulties in interpretability and security verification; Dynamic adaptation limitation	[196–199]
	RBF	1. It can quickly converge and improve training efficiency; Avoid local minima; Strong real-time 2. Complex network structure design; High-dimensional input performance degrades	[187, 190]
	GRNN	1. Fast training speed; High precision of nonlinear modeling; The performance is stable when the noise distribution is complex 2. High computational complexity and poor real-time performance; ECU memory consumption is high; Dynamic update capability is insufficient	[191–194]

conditions and vehicle dynamics. Comprehensive studies on TRFC, road roughness, road slope estimation, and vehicle state are still limited, and this research gap may restrict the overall accuracy of road condition identification and VSE. Effective integration of these factors would significantly improve the accuracy of road condition identification and the reliability of VSE. TRFC, road roughness, road slope, and vehicle state are closely related to each other, and analyzing only one of these factors alone may lead to a one-sided perception of the road condition, and even affect the adjustment of vehicle parameters. This limitation may lead to oversimplified judgments of the road environment, vehicle performance, and even driver behavior, which is not conducive to a comprehensive and accurate condition assessment. To improve the reliability of vehicle system estimates, it is recommended that future work explore the following key areas. They have great potential to improve the validity of these estimates and their implementation in realistic road situations.

1. The comprehensive identification of the road condition is essential to ensure an accurate assessment of the road properties. The different characteristics of the road surface: TRFC, road vertical excitation, and gradient are closely related to each other, and analyzing any one of these factors individually may lead to a one-sided understanding of the actual road conditions. Therefore, an integrated identification strategy can effectively integrate these key factors and improve the accuracy of road condition identification. The inseparability between road characteristics needs to be quantitatively analyzed by constructing an integrated vehicle and road model. These factors not only affect the road condition, but also directly determine the dynamic response and control strategy of the vehicle. Therefore, a modular approach must be used for road condition identification by comprehensively modeling the factors and their interactions and considering all the parameters that may be affected by them. Higher-quality data output not only optimizes road state identification, but also provides a stable base support for intelligent vehicle systems. The integrated identification approach can improve the adaptability of autonomous driving, intelligent obstacle avoidance, and vehicle dynamic control systems, enabling them to cope with fast-changing, complex, and volatile roadway environments, and improve overall driving safety and comfort.
2. The accuracy of the tire model is critical for road condition identification. Studies have shown that TRFC is a function of the interaction between road characteristics and tire characteristics; therefore, the quality of TRFC prediction relies heavily on the accuracy of the tire model. If the tire model is inaccurate, the theoretically calculated attachment coefficients will be biased, which in turn affects the reliability and efficiency

of vehicle control and safety systems. Accurate modeling of tire force characteristics is the key to ensuring the correct prediction of TRFC. Early studies relied heavily on experimental methods to establish tire force characteristic curves and their mathematical relationships from a large amount of test data. These experiments, which typically involve long automated tests, constant calibration, and data cycling for validation, are costly and time-consuming, albeit with high accuracy. A data-driven tire modeling approach provides a more adaptive solution for improving TRFC estimates. The method uses real-time sensor data to dynamically adapt the tire model to the complex and changing road environment. This approach is more robust to high-frequency changing conditions than the traditional static-parameter-based tire model, which helps to reduce errors and improve the accuracy of TRFC estimation. Therefore, the data-driven tire model becomes an important development direction to improve the accuracy of road condition identification.

3. The vertical load of tires is crucial for road condition identification, but it has been under-considered in most studies. The vertical load not only affects the interaction force between the tire and the road surface, but also determines the dynamic characteristics of the tire, which in turn affects the vehicle traction, stability control, and the accuracy of road condition identification. However, many existing studies fail to consider in detail the dynamic variation of vertical loads, which may lead to biased identification results. The variation of vertical loads is affected by a variety of factors, including changes in the vehicle body during steering, acceleration, braking, and under different road conditions and gradients. Due to the mechanical structural characteristics of the vehicle, the vertical force on the tires fluctuates with changing conditions, resulting in dynamic changes in road characteristics, slip rate, and adhesion coefficient. Such changes in turn affect the dynamic response of the vehicle, making it difficult for the traditional fixed-parameter-based tire model to accurately describe the actual working conditions. To improve the identification accuracy, the dynamic changes of vertical loads need to be correctly considered in the vehicle model. By constructing a vehicle simulation model under real working conditions, the influence of the vertical load on the tire characteristics can be described more accurately, and the estimation error can be reduced. In addition, by combining adaptive algorithms and real-time data supervision, the tire model can be continuously optimized so that it can adjust itself according to the changes in working conditions, thus improving the identification accuracy and enhancing the model's adaptability to complex road conditions.

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